An Inconsistent Linear System

Example 9: For the given linear system use elementary row operations to reduce the corresponding augmented matrix to row echelon form. Does the linear system have a solution? Explain.

$$\begin{cases} x_1 + 2x_2 - x_3 + x_4 = 5 \\ 2x_1 + 3x_2 - x_3 + 4x_4 = 8 \\ x_1 + 4x_2 - 3x_3 - 3x_4 = 6 \end{cases} \tag{1}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 5 \\ 2 & 3 & -1 & 4 & 8 \\ 1 & 4 & -3 & -3 & 6 \end{array}\right] \quad R_3 := R_3 - R_1, \; R_2 := R_2 - 2R_1 \quad \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 5 \\ 0 & -1 & 1 & 2 & -2 \\ 0 & 2 & -2 & -4 & 1 \end{array}\right]$$

$R_3 := R_3 + 2R_2$

$$\begin{array}{c} x_1\ x_2\ x_3\ x_4 \\ \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 5 \\ 0 & -1 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & -3 \end{array}\right] \end{array} \quad \Rightarrow \quad \begin{cases} x_1 + 2x_2 - x_3 + x_4 = 5 \\ -x_2 + x_3 + 2x_4 = -2 \\ 0 = -3 \; \text{※} \end{cases}$$

no solution

Thus the linear system (9) has no solutions.

(is inconsistent).